Technical Note

[Reconsideration of axial and radial resonance oscillations of acoustically levitated drops or particles for a reliable experimental concept]

[1. Objectives]

This technical note addresses users and potential users of acoustic standing wave levitators. It tries to promote the known [1], but seldom applied effect of low frequency axial and radial resonance oscillations of levitated drops and particles for an improved measuring- and experimenting concept. It allows a simple, precise and permanent calibration of acoustic levitators, independent of individual hardware and environmental conditions. It also allows a reliable process control during drop shrinkage or growth by evaporation or condensation and introduces low frequency sample accelerations up to 20 g_0 as new potential parameter.

Fig.1 shows a levitated spherical sample in an ultrasonic-standing wave levitator. Axial *radiation pressure* and radial *Bernoulli-forces,* both proportional to the *sound intensity* I_a of the radiating *transducer,* keep the drop suspended like a three-dimensional net of invisible, elastic filaments. The standing wave, schematically shown with 6 light areas of maximum sound pressure (*antinodes*) and 5 dark areas of minimum sound pressure (*nodes*) results from in-phase or counter-phase wave superpositions, caused by multiple reflections between the transducer and the concave solid reflector. The two components are coaxially aligned in a 5-half-wavelength ($5 \cdot \lambda_z / 2$) resonance distance. The optimal position for a levitated drop or particle is the shown area between the central pressure node and the supporting antinode underneath. The theory und practise of acoustic levitation are extensively described and discussed in the cited literature ([1] - [7]).

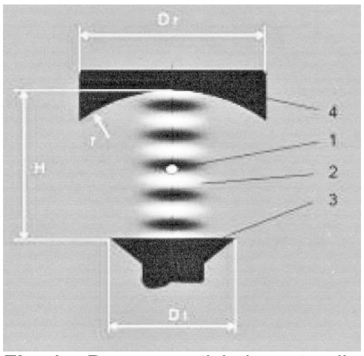

Fig. 1a: Drop or particle in a standing-wave-levitator (schematical)

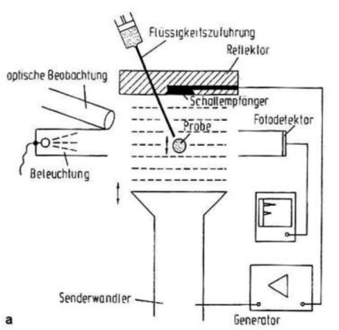

Fig .1b: Sample in standing-wave levitator with required hardware for resonance measurements

The proposed effect of low frequency, decaying *resonance* oscillations of liquid or solid samples can be observed, if the sample is slightly displaced from its stable position, either by manipulation with a tool from outside or by mechanical *shock impulses,* applied to the whole levitator. It results from the periodical exchange of the kinetic energy of the moving sample with the potential energy of the variable spring forces of the acoustic *levitation cage*.

Forced resonance oscillations of the sample can be generated, if the voltage of the levitating ultrasonic *carrier wave* is *amplitude modulated* with a low frequency voltage from an external generator (see fig.1b), which is connected with the respective entrance of the standard ultrasonic *power supply.* If this voltage is large enough and coincides in phase and frequency with the sample resonance, the sample will permanently oscillate.

A typical 58 kHz tec5 levitator with an axial to radial levitation force ratio of 3 :1 allows sample oscillations with axial frequencies between about 20 and 60 Hz, which increase with increasing acoustic power (see table 1). The measured axial and radial displacement amplitudes are limited to $\pm \lambda_z / 4$ and $\pm \lambda_z / 2$ respectively. The following sections will concentrate on axial oscillations.

[2. A few fundamental equations]

The weight $m_s g_0$ of the sample, in a stable equilibrium, is compensated by the axial levitation force $F_z = F_{z,max} \sin(4\pi \Delta z / \lambda_z) = m_s g_0$. The sample centre is axially downwards displaced by $\Delta z \le \lambda_z / 8$ from the central pressure node ($\Delta z = 0$). F_z acts like a spring under variable compression. The ratio of axial levitation force $F_{z,max}$ and sample weight $m_s g_0$ is called *levitation safety factor*

(1) $$\Phi_s = 1/\sin(4\pi \cdot \Delta z / \lambda_z) = F_{z,max} / m_s g_0 > 1$$

The resonance frequency f_0 of free axial sample oscillations (see appendix A3) follows from

(2a) $f_0 = f_{0,0} \cdot \sqrt[4]{\Phi_s^2 - 1}$ and (2b) $f_{0,0} = \sqrt{g_0 / \pi\lambda_z}$ with g_0 = 9.81 m/s^2.

The levitation force $F_{z,max}$ is a time-average, square function of the amplitude-modulated voltage $U = U_0 \sin(\omega_{us} t) + u_0 \sin(\omega_0 t)$, with the resonance frequency of the ultrasonic carrier wave, $f_{us} = \omega_{us} / 2\pi$ and the resonance frequency of the oscillating sample, $f_0 = \omega_0 / 2\pi$. Due to its inertia, the sample can only respond to the time average of the carrier wave and the low frequency component of U^2, which oscillates with $2f_0$. Thus the resulting effective equation for the amplitude modulation of the levitation force F_z is

(3) $\overline{F}_{z.max} \approx U_0^2 / 2 \cdot [1 + M \cdot (1 - \cos(2\omega_0 t))]$, with the modulation depth $M \approx (u_0 / U_0)^2$.

Because the response equation of the oscillating sample is a Mathieu-equation with a subharmonic solution [1], the sample will oscillate with f_o rather than with $2f_0$. This fits with the driving frequency f_0 of the modulation voltage and can be controlled by stroboscopic illumination.

Since $f_{0,0}$ in equation (2b) is constant at standard environmental conditions and constant ultrasonic frequency, the low frequency sample resonances f_0 are independent of density ρ_s and diameter d_s of the sample and depend solely on the levitation safety factor $\Phi_s > 1$.

The displacement amplitudes δz of the low frequency axial sample oscillations increase with increasing modulation depth M, but they are limited by the axial dimension of the levitation cage $\Delta z \le \pm \lambda_z / 4$ (cf. section 6).

For tec5 levitators we find under standard environmental conditions

at 58 kHz: λ_z = 6.6 mm, $f_{0,0}$ = 21.75 Hz and $\delta z < \pm 1.65$ mm
at 100 kHz: λ_z = 3.8 mm, $f_{0,0}$ = 28.55 Hz and $\delta z < \pm 0.95$ mm.

Table 1: Φ_s	1.5	**2.0**	2.5	3.0	3.5	4.0	4.5	5.0
$\overline{f} = f_0/f_{0.0}$	1.057	**1.316**	1.514	1.682	1.831	1.968	2.095	2.213
f_0/Hz (58 kHz)	23.0	**28.6**	32.9	36.6	39.8	42.8	45.6	48.1
f_0/Hz (100 kHz)	30.2	**37.6**	43.2	48.0	52.3	56.2	59.8	63.2

We recommend that experiments with levitated samples are performed at known, constant levitation safety factor Φ_s and resulting constant sample displacement Δz. This would allow a process control by a few simple measurements at one constant, low resonance frequency f_0 according to equations (2a,b) and table 1. It also allows a comparison of data from different authors.

[3. Calibration of the standing-wave levitator]

A reliable, permanent calibration requires amplitude and frequency measurements, which are independent of the environmental conditions, the electro-acoustic efficiency of the power supply, the resonance tuning of the levitator and also of the size, shape and density of the sample. This can not be guaranteed by the voltage or power readings at the front instrument of the standard power supply. A small piezoelectric sensor, cemented into a centric hole of the reflector (see Fig. 1b) is ideally suited for the calibration and tuning of the levitator. Its output voltage $U_{e,r}$ is exactly proportional to the square root of the levitation force $F_{z,max}$ at all pressure antinodes of the standing wave.

A solid sphere of exactly defined density $\rho_{s,ref}$ and diameter $d_{s,ref}$ is a reliable reference sample. We recommend a calibration at $\Phi_{s,ref} = 2$ ($\overline{f} = 1.316$), for instance with a polystyrene sphere with $\rho_{s,ref} = 1.05\, g/cm^3$ and $d_{s,ref} =$ 1mm (or 1.5 mm). After deployment of the calibration sphere in a 58 kHz or 100 kHz levitator at $\Phi_s > \Phi_{s,ref} = 2$ the ultrasonic power, modulated with f_0 = 28.6 Hz or 37.6 Hz, has to be carefully reduced, until the sample starts to oscillate. The respective calibration voltages $U_{e,ref}$ at the reflector sensor and the respective electric power $P_{e,ref}$ at the ultrasonic power supply have to be documented.

The levitation force $F_{z,max}(d_s, \rho_s)$ is proportional to the sample density ρ_s, the levitation safety factor Φ_s and the *sample size factor* f_1. This factor can be approximated (see appendix A1) with an accuracy deviation < $\pm 0.5\%$ by

(4) $$f_1(\overline{d}_s) \approx 1 + 0.61 \cdot P(c) \cdot \overline{d}_s^{\,2.36} \quad \text{with } \overline{d}_s = d_s / d_{s,opt} \text{ and } d_{s,opt} = \lambda_z / \pi .$$

P(c) is a *shape factor* for deformed drops with the aspect ratio c = a/b (see (A1.4)).
The following considerations are restricted to spherical drops, where c = P =1.

After normalization with $d_{s,ref}$ = 1mm, $\rho_{s,ref} = 1.05\, g/cm^3$ and $\Phi_{s,ref} = 2$ of the reference sample, we find from equation (4) the *calibration equations* for the 58 kHz und the 100 kHz tec5-levitator (see fig. 2):

(5a) $$\overline{U}_{e,1} = U_{e,1} / U_{e,ref,1} = \sqrt{0.435 \cdot \rho_s \cdot \Phi_s \cdot [1 + 0.61 \cdot (d_s / 2.1)^{2.36}]} \quad \text{with } d_s \le d_{s,opt,1} = 2.1 \text{ mm and } \rho_s / gcm^{-3}$$

(5b) $$\overline{U}_{e,2} = U_{e,2} / U_{e,ref,2} = \sqrt{0.343 \cdot \rho_s \cdot \Phi_s \cdot [1 + 0.61 \cdot (d_s / 1.21)^{2.36}]} \text{ with } d_s \le d_{s,opt,2} = 1.2 \text{ mm}$$

The reference values $P_{e,ref}$ and $U_{e,ref}$ for 58 kHz and 100 kHz are different, but measured with identical, 1mm calibration spheres of polystyrene.

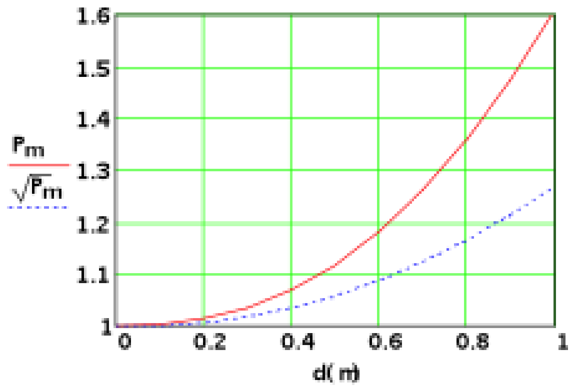

Fig. 2: Relative power and voltage requirement versus normalized sample diameter according to equations (5a,b)

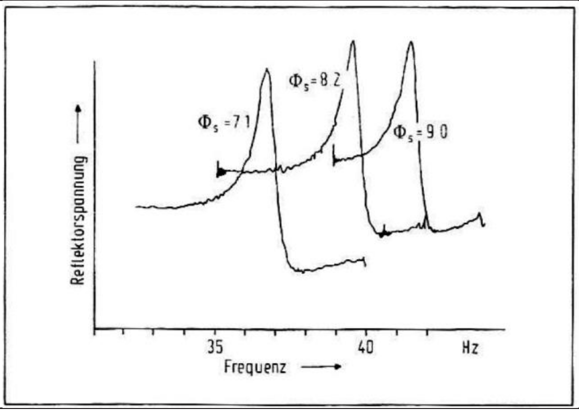

Fig. 3: Typical measured low frequency axial resonance curves according to [1], at different levitation safety-factor

Fig. 2:

The normalized ultrasonic power $\overline{P}_e$ and reflector the voltage $\overline{U}_e = \sqrt{\overline{P}_e}$ vary in the diameter range $0 \leq \overline{d}_s = d_s / d_{s,opt} \leq 1$ according to fig. 2 and equations (5a,b) between 1 and 1.6 and between 1 and $\sqrt{1.6}$ respectively. So the full range covers roughly 2dB. The sound pressure level (SPL) for the levitation of water drops at $\Phi_s = 2$ and 58 kHz would vary between about 155 and 157dB; at 100kHz between 152.6 and 154.6 db. Reference is 10^{-12} W/m^2 or $1W/cm^2$ = 160dB (appendix A2).

If the levitation force $F_{z,max}$ or the electric and acoustic power are kept constant during the variation of the drop diameter, the levitation safety factor Φ_s would vary like the upper curve in fig. 2.

[4. Control of the time slope of evaporating drops at constant] Φ_s

If a drop shrinks by evaporation under constant environmental conditions, its time dependent effective diameter follows the equation [4]

(6) $\overline{d}_s(t) = d_{s,0} / d_{s,opt} = \sqrt{1 - t/\tau}$, with τ being the *lifetime* for the complete evaporation.

In standard measurements this time function is based on optical determination of the cross section of the deformed drop with the half axes a, b and the aspect ratio c = a/b.

If we keep the safety factor Φ_s of the levitated drop constant during the evaporation, we can combine equations (5a,b) with (6) to get an estimated time slope, controllable by the reflector voltage U_e. The correct decrease of $U_e(t)$ can be monitored by periodically wobbling the modulation depth M and adjusting the carrier wave for constant f_0, = f(Φ_s). If the drop is deployed at an initial diameter $d_{s,0} \leq d_{s,p}$, we find two different normalised time functions for 58 kHz and 100 kHz respectively:

(7a) $U_e(\rho_s, d_s(t), \Phi_s)/U_{e,ref} = \sqrt{0.435 \cdot \rho_s \cdot \Phi_s \cdot [1 + 0.61 \cdot \sqrt{(d_{s,0}/2.1)^2 - t/\tau_{max})}^{2.36}]}$ at $d_{s,0} \leq 2.1$ mm

(7b) $U_e(\rho_s, d_s(t), \Phi_s)/U_{e,ref} = \sqrt{0.343 \cdot \rho_s \cdot \Phi_s \cdot [1 + 0.61 \cdot \sqrt{(d_{s,0}/1.21)^2 - t/\tau_{max}}^{2.36}]}$ at $d_s \leq 1.2$ mm.

The lifetime τ_{max} of the levitated drop with $d_{s,0} = d_{s,opt}$ is still unknown. However, according to [4], it is a known function of temperature T_∞, pressure p_0 and relative humidity φ_∞ of the surrounding gas. Under standard room conditions at p_0 = 1 bar and T_∞ = 20 °C we may use the approximation

(8) $\frac{\tau(d_{s,0})}{\min} \approx \frac{27.8/Nu}{(1-\phi_\infty)} \cdot (\frac{d_{s,0}}{mm})^2$, with $3 < Nu(\Phi_s) < 4$ as typical range for the Nusselt number.

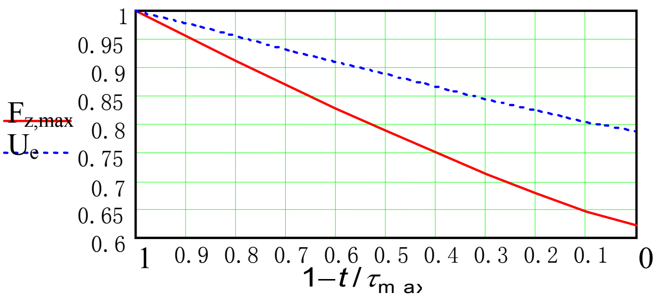

Fig. 4: Normalized levitation force and reflector voltage versus normalized time during drop evaporation according to equations (7a,b)

Table 2:

$1-t/\tau_{max}$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
$\bar{F}_{z,m}$	1	0.956	0.912	0.870	0.829	0.789	0.750	0.713	0.678	0.647	0.622
$\bar{U}_e$	1	0.978	0.955	0.933	0.910	0.888	0.866	0.844	0.824	0.804	0 788
d_s [mm]	2.1	1.99	1.88	1.76	1.63	1.49	1.33	1.15	0.94	0.66	0.0

Table 2 and fig. 3 show the normalized time slope of an evaporating water drop with the initial diameter $d_{s,0} = 2.1$mm in a 58 kHz levitator at constant levitation safety factor Φ_s. Reference data from the calibration with a 1mm polystyrene sphere at $\Phi_s = 2$ are used for normalization. Shown on the vertical axis is the normalized levitation force $\bar{U}_{e,r}^2 / 0.435\,\Phi_s \bar{\rho}_s$ (full line) and the normalized reflector voltage $\bar{U}_{e,r} / \sqrt{0.435 \cdot \Phi_s \cdot \bar{\rho}_s}$ (dotted line). Shown on the horizontal axis is the full normalized range $\bar{t}(d_{s,max}) = 1 - t/\tau_{max}$, for drops, deployed at $d_{s,0} = 2.1$mm. For smaller drops with $d_{s,0} < d_{s,max}$ the evaporation starts at t = 0 with $\bar{t} = (d_{s,0}/2.1)^2 - t/\tau_{max} \le 1$ and is described by the remaining range at the right side of fig. 3 and table 2. For the estimated τ_{max} (see equation (8)). In dry atmosphere ($\varphi_\infty = 0$) we get as estimation for the full diameter range $0 < d_{s,0} \le 2.1$mm: $30\text{ min} < \tau_{max} < 40\text{ min}$.

[5. Free resonance oscillations by sample feedback]

Axially oscillating samples have a position- and diameter-dependent feedback on the pressure antinode and the respective sensor voltage U_e at the reflector. A periodical axial up and down movement of the sample with f_0 will therefore result in a small respective modulation of the sensor voltage u_0. The feedback is smaller with the sample near the pressure nodes and larger near the pressure antinodes. We expect therefore a sensor voltage-modu-lation with f_0, which would fit with the requirement for forced excitation described in sect.2 and the proposed online sample-size monitoring as described in sections 3 and 4.

If we lead the small, low frequency feedback signal, after filtering and sufficient amplification, into the modulation entrance of the ultrasonic power supply, we expect - at least for relatively large samples - a permanent, self-generated resonance oscillation, without the necessity of an external low frequency generator. The drop oscillation can be stopped or periodically interrupted by suppressing the feedback signal u_0.

[6. Periodical accelerations of oscillating samples, a possible new parameter]

The axial displacement amplitudes δz of oscillating samples are limited by the tolerable displacement from the pressure node. If the static displacement Δz exceeds $\Delta z_{max} = \lambda_z/8$ the sample will be dropped from the levitation cage. However, the full range for dynamic axial sample movements is the distance $\pm\lambda_z/4$ between two pressure antinodes. Considering the displacement of the sample under its own weight, $\Delta z = -\lambda_z / 4\pi \cdot \arcsin(\Phi_s^{-1}) < \lambda_z/8$, we expect tolerable dynamic displacements $\delta_z < \delta_{z,max} = \lambda_z/8 \cdot (2 \pm 1/\pi \cdot \arcsin(\Phi_s^{-1}))$. This leads with equations (2a,b) to the normalized axial acceleration

(9a) $$a_z = \partial^2 \delta z / \partial t^2 = 4\pi^2 \cdot f_o^2 \cdot \delta z_m ,$$

(9b) $$\frac{a_z}{g_0} \le \sqrt{\Phi_s^2 - 1} \cdot [\pi \pm \arcsin(\Phi_s^{-1})] \approx \pi \cdot \sqrt{\Phi_s^2 - 1} \pm 1.$$

The periodical acceleration depends solely on the levitation safety factor Φ_s and is independent of the sample weight and size, of the environmental parameters and even of the ultrasonic frequency of the levitator.

Table 3 shows axial accelerations as function of the levitation safety factor Φ_s in the range $1.5 < \Phi_s < 7$, which reach up to about 22 g_0. This looks like an interesting range for experiments in materials science and in heat- and mass transfer with variable Nusselt numbers.

(*) Despite of the limit $-\lambda_z / 8$ for static displacements, dynamic axial displacements up to $\pm \lambda_z / 4$ and radial displacements up to $\pm \lambda_z / 2$have been measured at 20 kHz in [4] and would result in a_z/g_0 (*) according (9b).

Table 3:

Φ_s	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.6	7.0
a_z/g_0 (*)	3.5	5.4	7.2	8.9	10.5	12.2	13.8	15.4	17	18	20.2	21.8

[7. Conclusion and outlook]

We have tried to show, that the resonance-frequency-controlled levitation safety-factor $\Phi_s(f_0)$, is a dominant, representative parameter of an acoustic standing-wave-levitator.
It is useful for reliable and reproducible experimental investigations and measurements.

The described concept is based on a condensed theory with transparent assumptions. It has yet to be verified under different practical conditions concerning the physical properties of the sample and its environment. We recommend:

Verification and adaptation of the calibration equations (4) and (5a,b) with solid spherical samples of different diameters between about 0.1 mm and $\lambda_z / 2$.

Verification and adaptation of equations (4) and (5a,b) with deformed, liquid drops under consideration of equations (A1.3) and (A1.4) of appendix A1.

Verification of the concept of amplitude modulation of the ultrasonic standing wave and determination of the required modulation depth $M = u_0/U_0$ according to equation (3).

Inclusion of lateral (radial) sample oscillations in the experimenting concept.

If the physical and environmental parameters are known and the amplitude U_0 of the ultrasonic reflector voltage and the resonance frequency f_0 of the oscillating sample are measured, the calibration equations (4), (5a,b) and (A1.3) can be controlled and - if necessary – empirically corrected.

Literature:

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[6] Lierke, E. G.: Kontrollierte Massenänderungen von Tropfen in einem akustischen Stehwellen-Positionierer", Forschung im Ingenieurwesen (Engineering Research) Bd. 62 (1996), in German

[7] Pruppacher, H. R. and Klett, J. D.: Microphysics of clouds and precipitation", D. Reidel Publ. Comp. pp 440

Appendices:

A1. The drop sice and *shape factor* of spheroidally deformed drops

Equations (5) of [1] and (A6) of [2] describe the influence of the normalized sample size and shape parameter

(A1.1) $$X^2 = (k_z b)^2 + 0.5\cdot (k_r a)^2 = (k_z a_0)^2 \cdot \left[(b/a_0)^2 + 0.5\cdot (k_r/k_z)^2 \cdot (a/a_0)^2\right] \approx \bar{d}_s^2 \cdot (c^{-4/3} + 1/8\cdot c^{1/3})$$

on the required levitation force of a spheroidally deformed drop with the aspect ratio c = a/b ($a/a_0 = c^{1/3}$, $b/a_0 = c^{-2/3}$) in an acoustic standing wave with the wave number ratio $q = k_r/k_z = 0.5$.

The force factor

(A1.2) $$f_1^{-1}(X) = \frac{3}{(2X)^2}\cdot\left|\frac{\sin(2X)}{2X} - \cos(2X)\right|$$

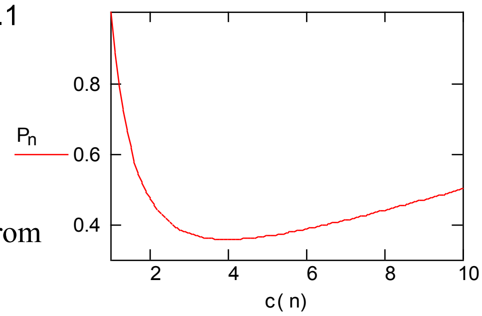
is a result of a pressure integration around the levitated sample. It can be approximated by

(A1.3) $$f_1(X) \approx 1 + 0.53\cdot X^{2.36} \approx 1 + 0.61\cdot P(c)\cdot \bar{d}_s^{2.36} \quad \pm 0.5\%.$$

The separated shape factor P(c) is shown in fig. A1.1, with P =1 for spherical samples (c = 1)

(A1.4) $$P(c) = \left[\frac{c^{-1.333} + 0.125\cdot c^{0.666}}{1.125}\right]^{1.18} \le 1$$

Fig.A1.1



A2. Mach number Ma and sound pressure level SPL

Equations (5) (6) and (12) with $k_r/k_z = 0.5$ lead to the required acoustic Mach number $Ma = v_{max}/c_0$ with

(A2.1) $$Ma^2 \approx 10^{-4}\cdot \frac{\lambda_z}{cm}\cdot \frac{\rho_s}{g/cm^3}\cdot \frac{\Phi_s}{p_0/at}\cdot f_1(\bar{d}_s, c)$$ (f_1 from (A1.3) and (A1.4))

and to the sound pressure level SPL with

(A2.2) $$SPL/dB = 10\cdot \log(\rho_0 c_0^3 \cdot I_{ref}^{-1}\cdot Ma^2),$$ reference is $I_{ref} \approx 10^{-12}$ W/m².

At standard room conditions we find:

58 kHz: $$Ma^2 \approx 0.67\cdot 10^{-4}\cdot \frac{\rho_s}{g/cm^3}\cdot \Phi_s \cdot [1 + 0.61\cdot (d_s/d_{s,opt})^{2.36}\cdot P(c)] \ , \qquad d_{s,opt} = \frac{\lambda_z}{\pi} = 2.1\,\text{mm}\ ,$$

$$SPL = 193.7 - 10\cdot \log(Ma^2) \approx 152 + 10\cdot \log(\rho_s \cdot \Phi_s \cdot [1 + 0.61\cdot (d_s/d_{s,opt})^{2.36}\cdot P(c)])$$ and

100 kHz: $$Ma^2 \approx 0.39\cdot 10^{-4}\cdot \frac{\rho_s}{g/cm^3}\cdot \Phi_s \cdot [1 + 0.61\cdot (d_s/d_{s,opt})^{2.36}\cdot P(c)] \qquad d_{s,opt} = \frac{\lambda_z}{\pi} = 1.2\,\text{mm}.$$

$$SPL = 193.7 - 10\cdot \log(Ma^2) \approx 149.6 + 10\cdot \log(\rho_s \cdot \Phi_s \cdot [1 + 0.61\cdot (d_s/d_{s,opt})^{2.36}\cdot P(c)])$$

A3. Calculation of the low axial resonance frequency f_0 :

Axial displacement : $z = \Delta z + \delta z \cdot \sin(\omega_0 t)$ with $\delta z / \Delta z < 1$

Equation of motion: $m_s \cdot \partial^2 z / \partial t^2 + F_{z,max} \cdot \sin(\alpha + \beta) = m_s g_0$

with $\alpha = 4\pi / \lambda_z \cdot \Delta z$, $\beta = 4\pi / \lambda_z \cdot \delta z \cdot \sin(\omega_0 t)$ and $(\sin\beta \approx \beta, \quad \cos\beta \approx 1)$

$$\partial^2 (\delta z \cdot \sin(\omega_0 t)) / \partial t^2 = -\omega_0^2 \cdot \delta z \cdot \sin(\omega_0 t) ,$$

separation: $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta \approx 1/\Phi_s + \sqrt{1 - 1/\Phi_s^2} \cdot 4\pi / \lambda_z \cdot \delta z \cdot \sin(\omega_0 t)]$

Static solution $\sin\alpha = 1/\Phi_s$, $F_{z,max} / m_s = \Phi_s \cdot g_0$, $\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - 1/\Phi_s^2}$

Low resonance frequency solution :

(A3.1) $\omega_0^2 \approx \Phi_s g_0 \cdot 4\pi / \lambda_z \cdot \sqrt{1 - 1/\Phi_s^2} = 4\pi g_0 / \lambda_z \cdot \sqrt{\Phi_s^2 - 1} \implies$ (2a,b) $f_0 \approx \sqrt{g_0 / \pi \lambda_z} \cdot \sqrt[4]{\Phi_s^2 -}$

Technology for Spectroscopy

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